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# PHILOSOPHICAL TRANSACTIONS.

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Monday, April 13. 1668

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## The Contents.

*The Squaring of the Hyperbola by an infinite series of Rational Numbers, together with its Demonstration, by the Right Honourable the Lord Viscount Brouncker. An Extract of a Letter sent from Danzick, touching some Chymical, Medicinal and Anatomical particulars. Two Letters, written by Dr. John Wallis to the Publisher; One, concerning the Variety of the Annual High-Tides in respect to several places: the other, concerning some Mistakes of a Book entituled SPECIMINA MATHEMATICA Francisci Dulaurens, especially touching a certain Probleme, affirm'd to have been propos'd by Dr. Wallis to the Mathematicians of all Europe, for a solution. An Account of some Observations concerning the true Time of the Tydes, by Mr. Hen. Philips. An Account of three Books: I. W. SENGERDIUS PH.D. de Tarantula. II. REGNERI de GRAEF M.D. Epistola de nonnullis circa Partes Genitales Inventis Novis. III. JOHANNIS van HORNE M.D. Observationum suarum circa Partes Genitales in utroque sexu, PRODROMUS.*

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*The Squaring of the Hyperbola, by an infinite series of Rational Numbers, together with its Demonstration, by that Eminent Mathematician, the Right Honourable the Lord Viscount Brouncker.*

**W**Hat the Acute Dr. John Wallis had intimated, some years since, in the Dedication of his Answer to M. Meibomius de proportionibus, vid. That the World one day would learn from the Noble Lord Brouncker, the *Quadrature of the Hyperbole*; the Ingenious Reader may see performed in the subjoyned operation, which its Excellent Author was now pleased to communicate, as followeth in his own words;

*My Method for Squaring the Hyperbola is this :*

**L**et AB be one *Asymptote* of the Hyperbola Ed C; and let AE and BC be parallel to th<sup>r</sup>other : Let also AE be to BC as 2 to 1; and let the Parallelogram ABDE equal 1. See Fig. 1. And note, that the Letter x every where stands for Multiplication.

Supposing the Reader knows, that EA.  $\alpha^2$ . KH.  $\beta$ n. d $\theta$ .  $\gamma$ x.  $\delta$ λ.  $\epsilon$ μ. CB.&c. are in an *Harmonic series*, or a *series reciproca primanorum seu arithmetice proportionalium* (otherwise he is referr'd for satisfaction to the 87, 88, 89, 90, 91, 92, 93, 94, 95, prop. *Arithm. Infinitor. Wallisij* :)

$$\left. \begin{aligned} \text{I say } ABCdEA &= \frac{1}{1 \times 2} + \frac{1}{3 \times 4} + \frac{1}{5 \times 6} + \frac{1}{7 \times 8} + \frac{1}{9 \times 10} \&c. \\ EdCDE &= \frac{1}{2 \times 3} + \frac{1}{4 \times 5} + \frac{1}{6 \times 7} + \frac{1}{8 \times 9} + \frac{1}{10 \times 11} \&c. \\ EdCyE &= \frac{1}{2 \times 3 \times 4} + \frac{1}{4 \times 5 \times 6} + \frac{1}{6 \times 7 \times 8} + \frac{1}{8 \times 9 \times 10} \&c. \end{aligned} \right\} \text{in infinitum.}$$

For (in Fig. 2, & 3) the Parallelog.

And (in Fig. 4.) the Triangl.

CA = $\frac{1}{1 \times 2}$	EdC = $\frac{1}{2 \times 3 \times 4} = \frac{\square dD - \square dF}{2}$	<p><i>Note.</i></p> <p>CA = dD + dF</p> <p>dD = br + bn</p> <p>dF = fG + fk</p> <p>br = aq + ap</p> <p>bn = cs + cm</p> <p>fG = et + el</p> <p>fk = gu + gh</p> <p>&amp;c.</p>
dD = $\frac{1}{2 \times 3}$   dF = $\frac{1}{3 \times 4}$	Ebd = $\frac{1}{4 \times 5 \times 6} = \frac{\square br - \square bn}{2}$	
br = $\frac{1}{4 \times 5}$   bn = $\frac{1}{5 \times 6}$	dfC = $\frac{1}{6 \times 7 \times 8} = \frac{\square fG - \square fk}{2}$	
fG = $\frac{1}{6 \times 7}$   fk = $\frac{1}{7 \times 8}$	Eab = $\frac{1}{8 \times 9 \times 10} = \frac{\square aq - \square ap}{2}$	
aq = $\frac{1}{8 \times 9}$   ap = $\frac{1}{9 \times 10}$	bcd = $\frac{1}{10 \times 11 \times 12} = \frac{\square cs - \square cm}{2}$	
cs = $\frac{1}{10 \times 11}$   cm = $\frac{1}{11 \times 12}$	def = $\frac{1}{12 \times 13 \times 14} = \frac{\square et - \square el}{2}$	
et = $\frac{1}{12 \times 13}$   el = $\frac{1}{13 \times 14}$	fgC = $\frac{1}{14 \times 15 \times 16} = \frac{\square gu - \square gh}{2}$	
gu = $\frac{1}{14 \times 15}$   gh = $\frac{1}{15 \times 16}$	&c.	
&c.	&c.	

And

And that therefore in the first series half the first term is greater than the sum of the two next, and half this sum of the second and third greater than the sum of the four next, and half the sum of those four greater than the sum of the next eight, &c. *in infinitum*. For  $\frac{1}{2} dD = br + bn$ ; but  $bn > fg$ , therefore  $\frac{1}{2} dD > br + fg$ , &c. And in the second series half the first term is less than the sum of the two next, and half this sum less than the sum of the four next, &c. *in infinitum*.

That the first *series* are the *even terms*, viz. the 2<sup>d</sup>, 4<sup>th</sup>, 6<sup>th</sup>, 8<sup>th</sup>, 10<sup>th</sup>, &c. and the second, the *odd*, viz. the 1<sup>st</sup>, 3<sup>d</sup>, 5<sup>th</sup>, 7<sup>th</sup>, 9<sup>th</sup>, &c. of the following series, viz.  $\frac{1}{1 \times 2} \cdot \frac{1}{2 \times 3} \cdot \frac{1}{3 \times 4} \cdot \frac{1}{4 \times 5} \cdot \frac{1}{5 \times 6} \cdot \frac{1}{6 \times 7} \cdot \frac{1}{7 \times 8} \cdot \frac{1}{8 \times 9} \cdot \frac{1}{9 \times 10} \cdot \frac{1}{10 \times 11} \cdot \frac{1}{11 \times 12} \cdot \frac{1}{12 \times 13} \cdot \frac{1}{13 \times 14} \cdot \frac{1}{14 \times 15} \cdot \frac{1}{15 \times 16} \cdot \frac{1}{16 \times 17} \cdot \frac{1}{17 \times 18} \cdot \frac{1}{18 \times 19} \cdot \frac{1}{19 \times 20} \cdot \frac{1}{20 \times 21} \cdot \frac{1}{21 \times 22} \cdot \frac{1}{22 \times 23} \cdot \frac{1}{23 \times 24} \cdot \frac{1}{24 \times 25} \cdot \frac{1}{25 \times 26} \cdot \frac{1}{26 \times 27} \cdot \frac{1}{27 \times 28} \cdot \frac{1}{28 \times 29} \cdot \frac{1}{29 \times 30} \cdot \frac{1}{30 \times 31} \cdot \frac{1}{31 \times 32} \cdot \frac{1}{32 \times 33} \cdot \frac{1}{33 \times 34} \cdot \frac{1}{34 \times 35} \cdot \frac{1}{35 \times 36} \cdot \frac{1}{36 \times 37} \cdot \frac{1}{37 \times 38} \cdot \frac{1}{38 \times 39} \cdot \frac{1}{39 \times 40} \cdot \frac{1}{40 \times 41} \cdot \frac{1}{41 \times 42} \cdot \frac{1}{42 \times 43} \cdot \frac{1}{43 \times 44} \cdot \frac{1}{44 \times 45} \cdot \frac{1}{45 \times 46} \cdot \frac{1}{46 \times 47} \cdot \frac{1}{47 \times 48} \cdot \frac{1}{48 \times 49} \cdot \frac{1}{49 \times 50} \cdot \frac{1}{50 \times 51} \cdot \frac{1}{51 \times 52} \cdot \frac{1}{52 \times 53} \cdot \frac{1}{53 \times 54} \cdot \frac{1}{54 \times 55} \cdot \frac{1}{55 \times 56} \cdot \frac{1}{56 \times 57} \cdot \frac{1}{57 \times 58} \cdot \frac{1}{58 \times 59} \cdot \frac{1}{59 \times 60} \cdot \frac{1}{60 \times 61} \cdot \frac{1}{61 \times 62} \cdot \frac{1}{62 \times 63} \cdot \frac{1}{63 \times 64} \cdot \frac{1}{64 \times 65} \cdot \frac{1}{65 \times 66} \cdot \frac{1}{66 \times 67} \cdot \frac{1}{67 \times 68} \cdot \frac{1}{68 \times 69} \cdot \frac{1}{69 \times 70} \cdot \frac{1}{70 \times 71} \cdot \frac{1}{71 \times 72} \cdot \frac{1}{72 \times 73} \cdot \frac{1}{73 \times 74} \cdot \frac{1}{74 \times 75} \cdot \frac{1}{75 \times 76} \cdot \frac{1}{76 \times 77} \cdot \frac{1}{77 \times 78} \cdot \frac{1}{78 \times 79} \cdot \frac{1}{79 \times 80} \cdot \frac{1}{80 \times 81} \cdot \frac{1}{81 \times 82} \cdot \frac{1}{82 \times 83} \cdot \frac{1}{83 \times 84} \cdot \frac{1}{84 \times 85} \cdot \frac{1}{85 \times 86} \cdot \frac{1}{86 \times 87} \cdot \frac{1}{87 \times 88} \cdot \frac{1}{88 \times 89} \cdot \frac{1}{89 \times 90} \cdot \frac{1}{90 \times 91} \cdot \frac{1}{91 \times 92} \cdot \frac{1}{92 \times 93} \cdot \frac{1}{93 \times 94} \cdot \frac{1}{94 \times 95} \cdot \frac{1}{95 \times 96} \cdot \frac{1}{96 \times 97} \cdot \frac{1}{97 \times 98} \cdot \frac{1}{98 \times 99} \cdot \frac{1}{99 \times 100} \cdot \frac{1}{100 \times 101} \cdot \frac{1}{101 \times 102} \cdot \frac{1}{102 \times 103} \cdot \frac{1}{103 \times 104} \cdot \frac{1}{104 \times 105} \cdot \frac{1}{105 \times 106} \cdot \frac{1}{106 \times 107} \cdot \frac{1}{107 \times 108} \cdot \frac{1}{108 \times 109} \cdot \frac{1}{109 \times 110} \cdot \frac{1}{110 \times 111} \cdot \frac{1}{111 \times 112} \cdot \frac{1}{112 \times 113} \cdot \frac{1}{113 \times 114} \cdot \frac{1}{114 \times 115} \cdot \frac{1}{115 \times 116} \cdot \frac{1}{116 \times 117} \cdot \frac{1}{117 \times 118} \cdot \frac{1}{118 \times 119} \cdot \frac{1}{119 \times 120} \cdot \frac{1}{120 \times 121} \cdot \frac{1}{121 \times 122} \cdot \frac{1}{122 \times 123} \cdot \frac{1}{123 \times 124} \cdot \frac{1}{124 \times 125} \cdot \frac{1}{125 \times 126} \cdot \frac{1}{126 \times 127} \cdot \frac{1}{127 \times 128} \cdot \frac{1}{128 \times 129} \cdot \frac{1}{129 \times 130} \cdot \frac{1}{130 \times 131} \cdot \frac{1}{131 \times 132} \cdot \frac{1}{132 \times 133} \cdot \frac{1}{133 \times 134} \cdot \frac{1}{134 \times 135} \cdot \frac{1}{135 \times 136} \cdot \frac{1}{136 \times 137} \cdot \frac{1}{137 \times 138} \cdot \frac{1}{138 \times 139} \cdot \frac{1}{139 \times 140} \cdot \frac{1}{140 \times 141} \cdot \frac{1}{141 \times 142} \cdot \frac{1}{142 \times 143} \cdot \frac{1}{143 \times 144} \cdot \frac{1}{144 \times 145} \cdot \frac{1}{145 \times 146} \cdot \frac{1}{146 \times 147} \cdot \frac{1}{147 \times 148} \cdot \frac{1}{148 \times 149} \cdot \frac{1}{149 \times 150} \cdot \frac{1}{150 \times 151} \cdot \frac{1}{151 \times 152} \cdot \frac{1}{152 \times 153} \cdot \frac{1}{153 \times 154} \cdot \frac{1}{154 \times 155} \cdot \frac{1}{155 \times 156} \cdot \frac{1}{156 \times 157} \cdot \frac{1}{157 \times 158} \cdot \frac{1}{158 \times 159} \cdot \frac{1}{159 \times 160} \cdot \frac{1}{160 \times 161} \cdot \frac{1}{161 \times 162} \cdot \frac{1}{162 \times 163} \cdot \frac{1}{163 \times 164} \cdot \frac{1}{164 \times 165} \cdot \frac{1}{165 \times 166} \cdot \frac{1}{166 \times 167} \cdot \frac{1}{167 \times 168} \cdot \frac{1}{168 \times 169} \cdot \frac{1}{169 \times 170} \cdot \frac{1}{170 \times 171} \cdot \frac{1}{171 \times 172} \cdot \frac{1}{172 \times 173} \cdot \frac{1}{173 \times 174} \cdot \frac{1}{174 \times 175} \cdot \frac{1}{175 \times 176} \cdot \frac{1}{176 \times 177} \cdot \frac{1}{177 \times 178} \cdot \frac{1}{178 \times 179} \cdot \frac{1}{179 \times 180} \cdot \frac{1}{180 \times 181} \cdot \frac{1}{181 \times 182} \cdot \frac{1}{182 \times 183} \cdot \frac{1}{183 \times 184} \cdot \frac{1}{184 \times 185} \cdot \frac{1}{185 \times 186} \cdot \frac{1}{186 \times 187} \cdot \frac{1}{187 \times 188} \cdot \frac{1}{188 \times 189} \cdot \frac{1}{189 \times 190} \cdot \frac{1}{190 \times 191} \cdot \frac{1}{191 \times 192} \cdot \frac{1}{192 \times 193} \cdot \frac{1}{193 \times 194} \cdot \frac{1}{194 \times 195} \cdot \frac{1}{195 \times 196} \cdot \frac{1}{196 \times 197} \cdot \frac{1}{197 \times 198} \cdot \frac{1}{198 \times 199} \cdot \frac{1}{199 \times 200} \cdot \frac{1}{200 \times 201} \cdot \frac{1}{201 \times 202} \cdot \frac{1}{202 \times 203} \cdot \frac{1}{203 \times 204} \cdot \frac{1}{204 \times 205} \cdot \frac{1}{205 \times 206} \cdot \frac{1}{206 \times 207} \cdot \frac{1}{207 \times 208} \cdot \frac{1}{208 \times 209} \cdot \frac{1}{209 \times 210} \cdot \frac{1}{210 \times 211} \cdot \frac{1}{211 \times 212} \cdot \frac{1}{212 \times 213} \cdot \frac{1}{213 \times 214} \cdot \frac{1}{214 \times 215} \cdot \frac{1}{215 \times 216} \cdot \frac{1}{216 \times 217} \cdot \frac{1}{217 \times 218} \cdot \frac{1}{218 \times 219} \cdot \frac{1}{219 \times 220} \cdot \frac{1}{220 \times 221} \cdot \frac{1}{221 \times 222} \cdot \frac{1}{222 \times 223} \cdot \frac{1}{223 \times 224} \cdot \frac{1}{224 \times 225} \cdot \frac{1}{225 \times 226} \cdot \frac{1}{226 \times 227} \cdot \frac{1}{227 \times 228} \cdot \frac{1}{228 \times 229} \cdot \frac{1}{229 \times 230} \cdot \frac{1}{230 \times 231} \cdot \frac{1}{231 \times 232} \cdot \frac{1}{232 \times 233} \cdot \frac{1}{233 \times 234} \cdot \frac{1}{234 \times 235} \cdot \frac{1}{235 \times 236} \cdot \frac{1}{236 \times 237} \cdot \frac{1}{237 \times 238} \cdot \frac{1}{238 \times 239} \cdot \frac{1}{239 \times 240} \cdot \frac{1}{240 \times 241} \cdot \frac{1}{241 \times 242} \cdot \frac{1}{242 \times 243} \cdot \frac{1}{243 \times 244} \cdot \frac{1}{244 \times 245} \cdot \frac{1}{245 \times 246} \$

That  $\frac{1}{4}$  of the first terme in the *third* series is less than the sum of the two next, and a quarter of this sum, less than the sum of the four next, and one fourth of this last sum less than the next eight, I thus demonstrate.

Let  $a$  = the 3<sup>d</sup> or last number of any term of the first Column, *viz.* of Divisors,

$$\frac{1}{2 \times \frac{1}{a-1} \times \frac{1}{a-2}} = \frac{1}{a^2 - 3a + 2a} = \frac{16a^3 - 48a^2 + 56a - 24}{16a^6 - 96a^5 + 232a^4 - 288a^3 + 184a^2 - 48a} = A$$

$$\left. \begin{aligned} \frac{2a}{x} - \frac{2a-1}{x} - \frac{2a-2}{x} &= \frac{I}{8a^3 - 12a^2 + 4a} \\ \frac{2a-2}{x} - \frac{2a-3}{x} - \frac{2a-4}{x} &= \frac{I}{8a^3 - 36a^2 + 52a - 24} \end{aligned} \right\} = \frac{16a^3 - 48a^2 + 56a - 24}{64a^6 - 384a^5 + 880a^4 - 960a^3 + 496a - 96} = B$$

$$\frac{64a^6 - 384a^5 + 928a^4 - 1152a^3 + 736a^2 - 192a}{64a^6 - 384a^5 + 880a^4 - 960a^3 + 496a^2 - 96a} x^{\frac{1}{2}} A < B.$$

And  $48a^4 - 192a^3 + 240a^2 - 96a =$  Excess of the Numerator above Denomina.

But — — The affirm.  $\begin{cases} 48a^3 + 240a^2 \\ a^3 + 5a^2 \\ a^3 + 5a \end{cases}$  the Negat.  $\begin{cases} 192a^3 + 96a \\ 4a^3 + 2a \\ 4a^3 + 2 \end{cases}$  if  $a > 2$ .

Therefore  $B \geq \frac{1}{4} A$ .

Therefore, of any number of A: or Terms, is less than their so many respective B. that is, than twice so many of the next Terms. *Quod, &c.*

Bv

By any one of which three Series, it is not hard to calculate, as near as you please, these and the like *Hyperbolic* spaces, whatever be the *Rational* Proportion of *A E* to *B C*. As for Example, when *A E* is to *B C*, as 5 to 4. (whereof the Calculation follows after that where the Proportion is, as 2 to 1. and both by the third Series.)

First then when (in Fig. 1.)  $A E, B C :: 2. 1.$

2 x 3 x 4) I. (0.0416666666—	0.0416666666
4 x 5 x 6) I. (0.0083333333—	0.0113095237
6 x 7 x 8) I. (0.0029761904—	
8 x 9 x 10) I. (0.0013888888—	0.0029019589
10 x 11 x 12) I. (0.0007575757—	
12 x 13 x 14) I. (0.0004578754—	
14 x 15 x 16) I. (0.0002976190—	
16 x 17 x 18) I. (0.0002042484—	0.0007306482
18 x 19 x 20) I. (0.0001461988—	
20 x 21 x 22) I. (0.0001082251—	
22 x 23 x 24) I. (0.0000823452—	
24 x 25 x 26) I. (0.0000641026—	
26 x 27 x 28) I. (0.0000508751—	
28 x 29 x 30) I. (0.0000410509—	
30 x 31 x 32) I. (0.0000336021—	
32 x 33 x 34) I. (0.0000278520—	0.0416666666
34 x 35 x 36) I. (0.0000233426—	0.0113095237
36 x 37 x 38) I. (0.0000197566—	0.0029019589
38 x 39 x 40) I. (0.0000168691—	0.0007306482
40 x 41 x 42) I. (0.0000145180—	3) 0.0001829939 (0.0000609980
42 x 43 x 44) I. (0.0000125843—	0.05679179
44 x 45 x 46) I. (0.0000109793—	+ 0.00006100
46 x 47 x 48) I. (0.0000096361—	0.05685279 < Ed Cy
48 x 49 x 50) I. (0.0000085034—	But 0.0007306482 } 0.0001829939 } 0.0000458315 }
50 x 51 x 52) I. (0.0000075415—	
52 x 53 x 54) I. (0.0000067193—	
54 x 55 x 56) I. (0.0000060125—	
56 x 57 x 58) I. (0.0000054014—	Therefore 0.05679179 + 0.00004583 + 0.00001528 0.05685290 > Ed Cy.
58 x 59 x 60) I. (0.0000048704—	
60 x 61 x 62) I. (0.0000044068—	
62 x 63 x 64) I. (0.0000040002—	

For, it has been demonstrated that, of any terme in the last Column is less than the terme next after it; and therefore that, of the last terme, at which you stop

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top, is less than the remaining terms, and that the total of these is less than  $\frac{1}{3}$  of a third proportional to the two last.

And therefore ABCyE being =  $0.75$  —————  $0.75$   
and Ed Cy >  $0.05685279$  ————— and <  $0.05685290$

And ABCdE is <  $0.69314720$  ————— and >  $0.69314709$

But when AE . BC :: 5 . 4. or as EA. to KH. then will the space ABCE. or now, the space AHKE (AH =  $\frac{1}{4}$ AB.) be found as follows.

8 x 9x10) 1 (0.0013888888	0.00 3888888
16x17x18) 1 (0.0002042484	0.0003504472
18x19x20) 1 (0.0001461988	3) 0.0000878204 (0.0000292735
32x33x34) 1 (0.0000278520	0.001871564
34x35x36) 1 (0.0000233426	0.0000292735
36x37x38) 1 (0.0000197566	0.0018564299 < Eab
38x39x40) 1 (0.0000168691	

But 0.0003504472 }  
0.0000878204 }  
0.00002200737 }

Therefore 0.0018271564  
+ 0.0000220074  
+ 0.0000073358  
0.0018564996 > Eab

Therefore EMb. (Fig 4.)

being =  $0.025$  —————  $0.025$   
Eab >  $0.0018564299$  ————— & <  $0.0018564996$

EMba (Fig. 4.) or EKM (Fig. 1.) >  $0.02685643$  ————— <  $0.02685650$   
AHKM <  $0.22314356$  ————— >  $0.22314349$

Therefore 3 ABCdE =  $2.07944154$   
and AHKE =  $0.2231435$  —————  
ABCdE (when AE.BC :: 10.1.) =  $2.025850$  —————

Therefore the Logar. of 10 $\frac{1}{3}$   
is to the Log. of 2,  
as 2.302585  
to 0.693147

An

